Model for the interaction of poloidal flow and magnetic islands in LHD

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Theses

- An interpretation of the observed ‘healing’ of the m/n = 1/1 vacuum magnetic island on LHD as being due to the properties of the plasma flow.
  - Reminiscent of ‘mode locking/unlocking’ physics of tokamak/RFP physics with two asymptotic states
    - Large nonrotating island locked to an external field error source
    - Rotational suppression of vacuum island --- eddy currents flowing at the rational surface shield the external field error from penetrating
  - Transitions between these two asymptotic states are described by coupled torque balance and island evolution equations.

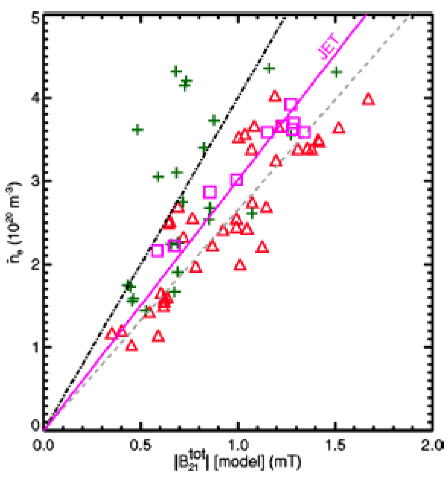
Outline

- Background
- Magnetic island topology
- Torque balance
- Unlocking a locked island
- Mode penetration into a rotating plasma
- Interpretation of LHD’s island physics
- Summary

In tokamaks and RFPs, mode locking physics has been widely studied

- In tokamaks, two classes of problems arise
  - Mode penetration into a rotating plasma ---> Resonant field errors producing a forced reconnection at the mode rational surface. Causes an abrupt change in the rotation profile.

    ---> Mode locking threshold $B_{2/1} \sim n$



•Mode locking of a pre-existing rotating magnetic island ---> Island rotation frequency abruptly drops from ~10's of kHz to 0. Often leads to disruptions.

Wolfe et al PoP '05
Comparison of JET and CMOD

Theoretical paradigm for understanding mode locking phenomena has been developed

- Amplitude and phase of magnetic island is determined by coupled electromagnetic and fluid flow information (Fitzpatrick, NF ‘93, …)
  - Asymptotic matched tearing layer solutions (or modified Rutherford theory for nonlinear islands)
  - Torque balance

    ---> Bifurcation theory
- Predictions for ‘locking’ threshold - transition from a flowing plasma with no island to a locked state with a large island
  - ‘unlocking’ threshold - reverse transition

  ---> Hysteresis

$$\tilde{B}_{r,locking} > \tilde{B}_{r,unlocking}$$

Island producing magnetic fields is distinguished from the equilibrium field

- The vacuum magnetic field is separated into fields that have topologically toroidal magnetic surfaces and island producing field

$$\vec{B} = \vec{B}_0 + \vec{B}_1 \qquad \vec{B}_0 \cdot \nabla \psi_0 = 0, \quad \vec{B}_1 \cdot \nabla \psi_0 \,|_{q=m_o/n_o} = \tilde{\psi}_1 \cos(m_0\theta - n_0\zeta)$$

  - Island producing magnetic topology

$$\vec{B} \cdot \nabla \Psi^* = 0,$$

$$\Psi^* = \iota_o' \frac{x^2}{2} - \tilde{\psi}_1 \cos(n_o\zeta - m_o\theta)$$

Island width = w

$$w = 4\sqrt{\frac{|\tilde{\psi}_1|}{|\iota_o'|}}$$

  - In tokamaks,
    - **B**$_O$ = Axisymmetric equilibrium
    - **B**$_1$ = Error or applied 3-D field with resonant component
  - For stellarators,
    - **B**$_O$ = Equilibrium with topological magnetic surfaces,
    - **B**$_1$ = Error or applied 3-D field with resonant component

In the absence of a plasma response, the island width is determined by the vacuum field

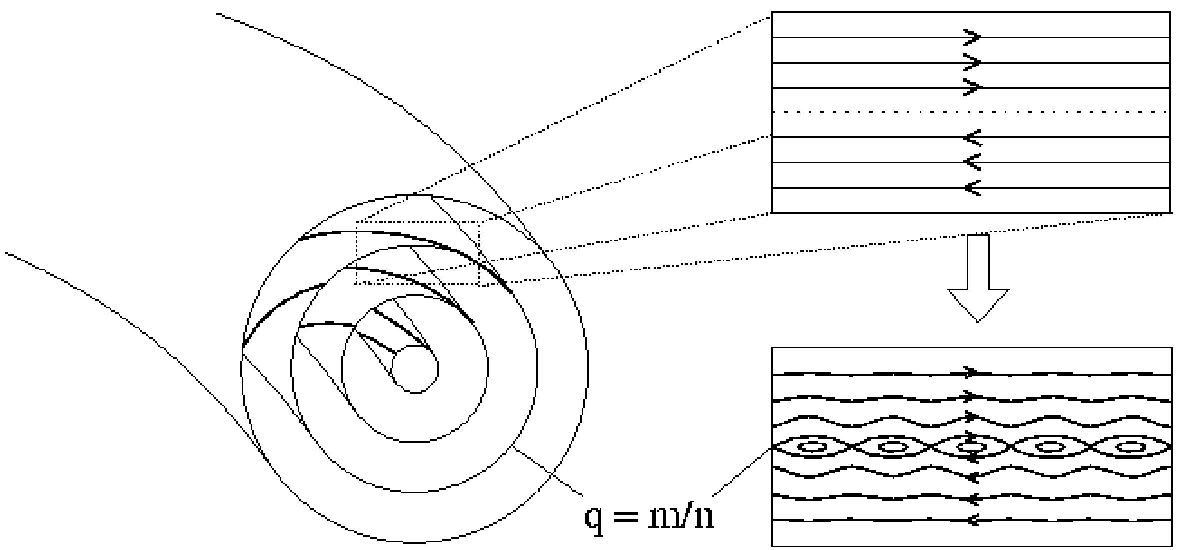
- With a vacuum response, a unique value of the island producing magnetic field is determine. Denote ψ^v as the vacuum value.

$$w^v = 4\sqrt{\frac{|\psi^v|}{|\iota'|}}$$

$$w = w^v$$

- With plasma response, generally

$$w \neq w^v$$



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As finite β plasmas are produced, generally the plasma flow rises

- In response to applied torques (I. e., NBI) or as a result of self-consistently general radial electric fields, plasmas flow with finite β.

$$\rho \frac{d\vec{v}}{dt} = \vec{J} \times \vec{B} - \nabla p - \sum_s \nabla \cdot \vec{\vec{\pi}}_s + S$$

  - On t > t_{MHD}, MHD force balance J x B =Grad p and flows within magnetic surfaces.

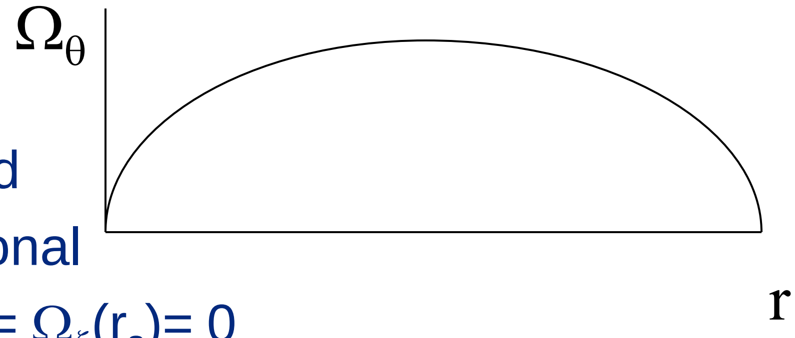
$$\vec{E} + \vec{v}_s \times \vec{B} = \frac{\nabla p_s}{n_s q_s}$$

  - On longer timescale, transport effects control rotation profiles
  - Generally, we would expect the rotation amplitudes to rise monotonically with plasmas β.

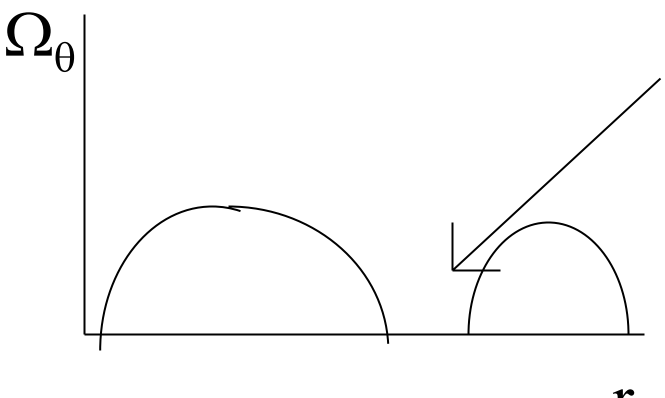
The presence of a locked island alters the rotation profile

- In the absence of a locked island, one can solve for the equilibrium flow profile (cylindrical approximation, for simplicity)

$$\vec{v} = \Omega_\theta \vec{e}_\theta + \Omega_\zeta \vec{e}_\zeta$$



- In the presence of a locked island, the rotation at the rational surface is fixed to zero $\Omega_\theta(r_s) = \Omega_\zeta(r_s) = 0$



Island region of width w with $\Omega_\theta = 0$
---> Presence of locked island produces $\Delta\Omega_\theta$ and $\Delta\Omega_\zeta$ profiles.

In the presence of cross-field viscosity, the presence of a locked island produces a localized viscous torque

- To model the plasma torque balance equations at the rational surface, a phenomenlogical cross-field viscosity is used

$$-\nabla \cdot \overleftrightarrow{\pi}_s = \nabla \cdot (\rho \nu_\perp \nabla \vec{v}) + ...$$

  – In general, the first derivatives of the flow profiles are discontinuous across the island region. This fact, in combination with a cross-field viscosity, produces a viscous torque at the rational surface

$$T_{VS\theta s} = \int_{r_s - w/2}^{r_s + w/2} dr \int dS \vec{e}_\theta \cdot \nabla \cdot (\rho \nu_\perp \nabla \vec{v}) = 4\pi^2 R_o \rho \nu_\perp r_s^3 \frac{\partial \Delta\Omega_\theta}{\partial r} \Big|_{r_s - w/2}^{r_s + w/2}$$

$$T_{VS\zeta s} = \int_{r_s - w/2}^{r_s + w/2} dr \int dS \vec{e}_\zeta \cdot \nabla \cdot (\rho \nu_\perp \nabla \vec{v}) = 4\pi^2 R_o^3 \rho \nu_\perp r_s \frac{\partial \Delta\Omega_\zeta}{\partial r} \Big|_{r_s - w/2}^{r_s + w/2}$$

The localized viscous torque is balance by a localized electromagnetic torque

- The interaction of a magnetic island and a resonant magnetic perturbation generally produces torques in the plasmas localized to the rational surface.

$$T_{EM\theta}(r) = \int dS \vec{e}_\theta \cdot \tilde{\vec{J}} \times \tilde{\vec{B}} \cong T_{EM\theta s}\delta(r - r_s)$$

$$T_{EM\zeta}(r) = \int dS \vec{e}_\zeta \cdot \tilde{\vec{J}} \times \tilde{\vec{B}} \cong T_{EM\zeta s}\delta(r - r_s)$$

  – In steady state, viscous and electromagnetic torques balance $T_{VS\theta s} + T_{EM\theta s} = 0$

$$T_{VS\zeta s} + T_{EM\zeta s} = 0$$

    - Due to $T_{EM\theta s} = -(m_o/n_o)T_{EM\zeta s}$

$$T_{VS\theta s} + \frac{m_o}{n_o} T_{VS\zeta s} = 0,$$

$$\frac{\partial \Delta\Omega_\theta}{\partial r}\Big|_{r_s-}^{r_s+} = -\frac{m_o R_o^2}{n_o r_s^2} \frac{\partial \Delta\Omega_\zeta}{\partial r}\Big|_{r_s-}^{r_s+} \quad \Rightarrow \frac{\partial \Delta\Omega_\theta}{\partial r}\Big|_{r_s-}^{r_s+} >> \frac{\partial \Delta\Omega_\zeta}{\partial r}\Big|_{r_s-}^{r_s+}$$

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Eddy currents at the rational surface modify island widths and produce localized EM torques

- Consider the case of an initial configuration with a large vacuum magnetic island. In order for the island to be maintained at the level given by the initial vacuum field, it must be locked to the phase of the external field.
  - Assuming the 'no-slip' condition (the island cannot slip through the plasma), plasma rotation at $r = r_s$ is inhibited.
  - Eddy currents at the rational surface develop. The out-of-phase part of the current produces a net **J** X **B** force.
  - The in-phase part of the currents modify the stability properties of the island.

$$\tilde{\psi}_1(r_s) = \tilde{\psi}^V \cos(\Delta\phi)$$

$$w = w^V \sqrt{\cos(\Delta\phi)}$$

$\Delta\phi$ = Phase difference between Island producing field and 3-D vacuum field

  - Finite-β effects (e. g. Bhattacharjee et al '95) ignored for simplicity.

The out-of-phase eddy currents produce a localized electromagnetic torque

- Using the calculated eddy current response

$$\tilde{J}_{eddy} \sim \psi^V \quad \tilde{B} \sim \psi = \psi^V \cos(\Delta\phi)$$

$$T_{EM\theta s} = -4\pi^2 R_o \frac{m_o^2}{\mu_o} |\tilde{\psi}_1||\psi^V| \sin(\Delta\phi)$$

  – From torque balance $T_{VS\theta s} + T_{EM\theta s} = 0$.

$$D_\Omega = D_w \sin(2\Delta\phi)$$

$$D_\Omega \equiv r_s \frac{\partial \Delta\Omega_\theta}{\partial r}\Big|_{r_s-}^{r_s+}$$

$$D_w \equiv \frac{\omega_A^2 \tau_V n_0^2 \hat{s}^2}{512}\left(\frac{w^V}{r_s}\right)^4 \qquad \tau_V = \frac{r_s^2}{\nu_\perp} \quad \omega_A^2 = \frac{B_o^2}{\mu_o \rho_o R_o^2}$$

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The torque balance equation predicts a critical condition for mode unlocking

- For small viscous torque, torque balance predicts a differential phase between the island producing field and the vacuum field

$$\sin(2\Delta\phi) = \frac{D_\Omega}{D_W}$$

Torque balance determines differential phase

$$w = w^V \sqrt{\cos\Delta\phi}$$

Phase shift reduces island width

$$D_\Omega \equiv r_s \frac{\partial \Delta\Omega_\theta}{\partial r}\Big|_{r_s-}^{r_s+} \quad D_w \equiv \frac{\omega_A^2 \tau_V n_0^2 \hat{s}^2}{512}\left(\frac{w^V}{r_s}\right)^4 \sim (B^V)^2$$

  - At the critical point, $D_\Omega = D_w$, $\sin(2\Delta\phi) = 1$, $w = w^V/2^{1/4}$
  - At higher β (higher rotation, D_Ω), the viscous torque overwhelms the electromagnetic torque ----> plasma at $r = r_s$ starts to rotate, the island is no longer locked to the vacuum field, the island width decays to a very small value.
  - The 'unlocking' threshold is given by

$$D_\Omega = D_w$$

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In a rotating plasmas, eddy currents at the rational surface prevent the vacuum fields from penetrating

- In a rotating plasma, the island width is suppressed. The island producing magnetic field is given by the expression

$$\tilde{\psi}_1 = \frac{\psi^V}{1+\delta}$$

  - The "1" in the denominator is the vacuum response
  - The "δ" represents eddy currents flowing in the layer
    - The model for the layer depends upon the sophisication of the model, but generally it has the form

$$\delta = i\omega\tau_L$$

$$\omega = \vec{k}\cdot\vec{v} = m_o\Omega_\theta - n_o\Omega_\zeta$$

τ_L = resistive layer timescale. Generally $\omega\tau_L$ >> 1

--> Rotation inhibits the penetration of the vacuum field.

Torque balance generally provides multiple solutions for the rotation velocity

- Amplitude and differential phase

$$|\tilde{\psi}_1| = \frac{|\psi^V|}{\sqrt{1+\omega^2\tau_L^2}} \qquad \sin(\Delta\phi) = \frac{\omega\tau_L}{\sqrt{1+\omega^2\tau_L^2}}$$

  - Torque balance equation

$$D_\Omega = D_w \frac{2\omega\tau_L}{1+\omega^2\tau_L^2}$$

  - Further approximation, D_Ω linearly depends on the difference between the frequency (ω) and the 'natural frequency' (ω_o) the value of the frequency in the absence of the island

$$D_\Omega = d_\Omega(\omega_o - \omega) \qquad \omega_o = (m_o\Omega_\theta - n_o\Omega_\zeta)_{no\ island}$$

$$\Rightarrow d_\Omega(\omega_o - \omega) = D_w \frac{2\omega\tau_L}{1+\omega^2\tau_L^2}$$

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Torque balance predicts bifurcation process between asysmptotic states

- Torque balance has 3 solutions for ω□$m_o\Omega_\theta$ - $n_o\Omega_\zeta$

$$d_\Omega(\omega_o - \omega) = D_w \frac{2\omega\tau_L}{1+\omega^2\tau_L^2}$$

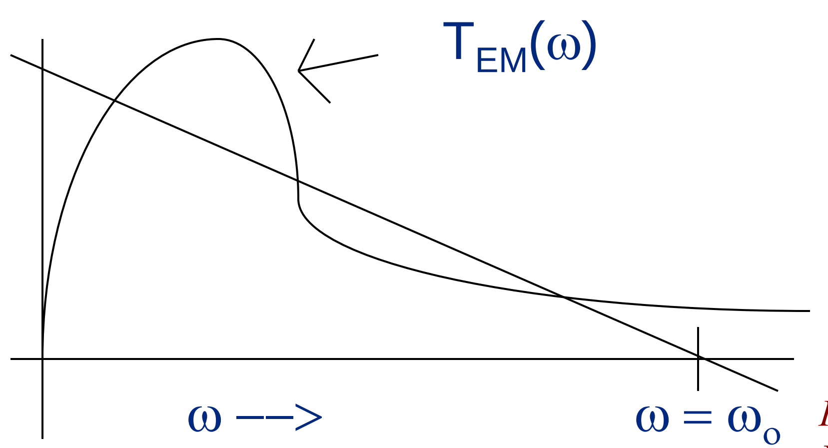
Asymptotic solutions:

– D_W small, only solution $\omega = \omega_o$ (large rotation, small island)

– D_W large, only solution $\omega = 0$ (small rotation, large island)

– Third root is dynamically unstable (not realizable)

In general



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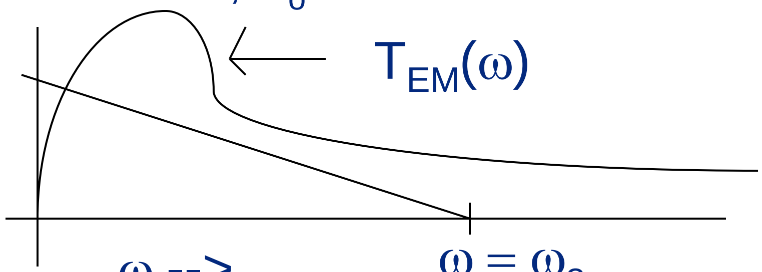
At lower ω_o, only one solution is allowed

- As β is lowered, ω_o decreases



  - Only the small rotation root survives ---> Abrupt transition from the high rotation/no island state to the low rotation/large island state
    - "Locking" threshold when

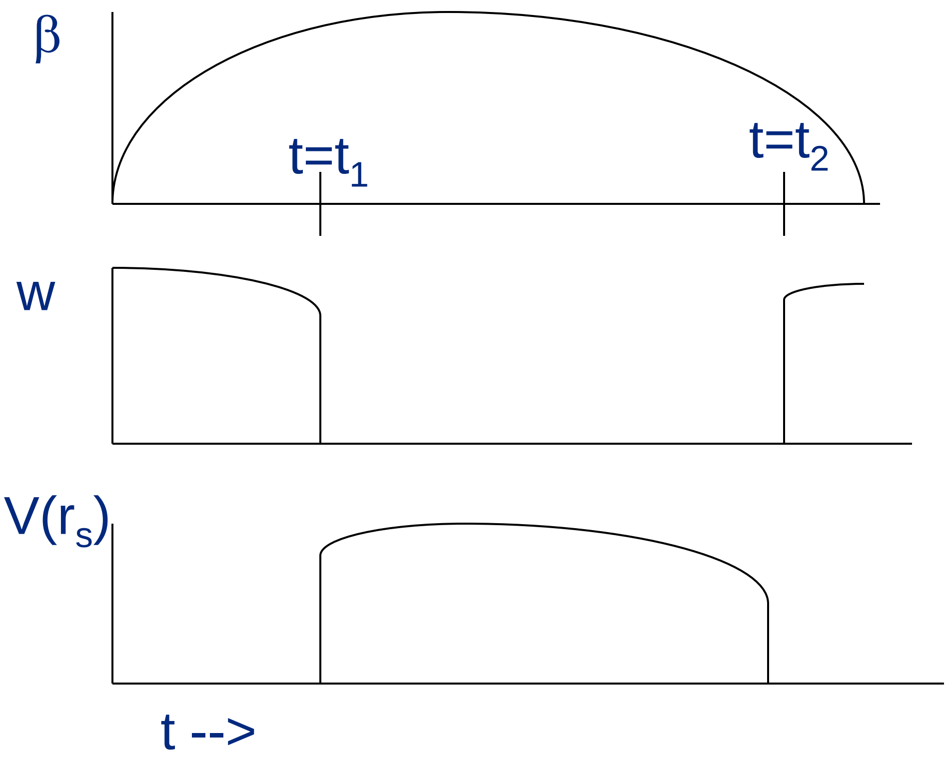
$$\omega_o = \omega_{oC} = \sqrt{\frac{8D_W}{d_\Omega \tau_L}}\left[1 + O\left(\frac{1}{\omega_o^2 \tau_L^2}\right)\right]$$

    - At locking threshold, $D_\Omega < D_W$ ---> Hysteresis

$$D_\Omega = d_\Omega(\omega_o - \omega) \cong \frac{D_W}{\omega \tau_L} << D_W$$

An interpretation of observations on LHD can be made using theory

- Evolution of the stored energy, island width and rotation at r_s
- At t = 0, large vacuum
Island
- For $0 < t < t_1$ viscous
Torque raises with β
- At $t = t_1$, $D_w = D_\Omega$
Plasma abruptly jumps
To rotating state at r_s
- For $t_1 < t < t_2$ flow shields
Island formation
- At $t = t_2$ plasma abruptly
Jumps to the low rotation/
Large island state at r_s



Conclusions

- A theory accounting for the evolution of the magnetic island width and phase is obtained from coupled electromagnetic and fluid flow information.
  - Theory is remiscient of mode locking/unlocking theory from tokamak physics.
  - Unlike axisymmetric equilibria, neoclassical damping physics is important for both the toroidal and poloidal flows.
  - Generally, the locking criteria differs from unlocking criteria. ---> Hysteresis in the flow/island width evolution.
  - Simple island physics model used in the calculation. Finite-β corrections to island physics should be included
- Implication: Plasma rotation physics can play a crucial role in healing magnetic islands. This physics is not incorporated in 3-D MHD equilibrium codes (PIES, SIESTA, etc). However, extended MHD codes should be able to include this physics.