Electromagnetic gyrokinetic turbulence in finite-beta helical plasmas

(New saturation mechanism of turbulence in the presence of 3D magnetic field)

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GK simulations of LHD plasmas

- Interplay between ITG turbulence and zonal flow in Large Helical Device (LHD)

  ![Image of LHD plasmas](image)


- Validation of gyrokinetic simulations against ITG turbulence in LHD experiments

  ![Image of gyrokinetic simulations](image)

  Nunami et al. PoP 2012

- Previous GK simulations of LHD plasmas assumed adiabatic electron response. Turbulent transport problem in finite beta helical plasma has not been previously explored because of numerical difficulties.
Saturation problem in finite beta plasmas

Failure of the transport levels to saturate at finite beta in gyrokinetic simulations in flux tube geometry

Cyclone base case (tokamak)

Finite beta (ITG) Runaway above $\beta_e = 0.75\%$

Finite beta (ITG) $\beta = 0.9\%$


Zonal flows are weak.
A new structure formation is expected.
Structure formation that affects saturation of instabilities

- Low beta: ITG mode
  - Stabilized by zonal flow structure.

- High beta: Ballooning (MHD) instability
  - Acceleration by finger-like structure

Mode structures of ITG and KBM are similar. Both of them have ballooning structure in the linear growth.
New saturation mechanism

• In this work, we present that a finite beta turbulence is saturated by a new mechanism.
• The mechanism originates from the three-dimensionality of magnetic field configuration.

EM delta-f gyrokinetic equations

\[
\frac{D\delta f_{sk}}{Dt} + \nu_{Ts} v_{\parallel} b^* \cdot \nabla \delta f_{sk} - \nu_{Ts} \mu b \cdot \nabla B \frac{\partial \delta f_{sk}}{\partial v_{\parallel}} = -iv_{ds} \cdot k_{\perp} (\delta f_{sk} + \frac{q_s F_{SM}}{T_s} \phi_k J_{0s}) \\
+ iv_s \cdot k_{\perp} \frac{q_s F_{SM}}{T_s} (\phi_k - \nu_{Ts} v_{\parallel} A_{\parallel k}) J_{0s} + \nu_{Ts} v_{\parallel} \frac{q_s F_{SM}}{T_s} E_{\parallel k} + C(\delta f_{sk}) \\
\lambda^2_{Dk} k_{\perp}^2 \phi_k = \sum_s \left( q_s \delta n_{sk} - \frac{q_s^2}{T_s} [1 - \Gamma_{0s}] \phi_k \right) \quad k_{\perp}^2 A_{\parallel k} = \beta_i \sum_s q_s \delta u_{sk}
\]

\[
E_{\parallel k} = -b^* \cdot \nabla \phi_k J_{0s} - \frac{\partial A_{\parallel k}}{\partial t} J_{0s}
\]

\[
\delta n_{sk} = \int dv^3 \delta f_{sk} J_{0s}
\]

\[
\delta u_{sk} = \int dv^3 v_{\parallel} \delta f_{sk} J_{0s}
\]

\[
\frac{D}{Dt} = \frac{\partial}{\partial t} + \frac{1}{B} [\phi J_{0s}, ]_k
\]

\[
b^* \cdot \nabla = b \cdot \nabla - \frac{1}{B} [A_{\parallel} J_{0s}, ]_k
\]
Linear analysis of LHD#88343
High Ti discharge LHD 88343

- $B_0=2.75\text{T}$, $R_{ax}=3.6\text{m}$ (shifted to 3.75m)
- Beta($r/a=0.65$)=0.3%
- banana regime

- Unstable against ITG mode
- KBM is unstable at high beta with the same configuration
- KBM with finite radial wave-number (red) is more unstable than that without it (blue).
The profile of electrostatic potential along the field line

LHD High-Ti \( \rho=0.65 \) \( \eta_i=\eta_e \) \( \beta=0.2\% \)

- \( k_y \rho=0.3 \) (red)
- \( k_y \rho=0.6 \) (blue)

Graphs showing the electrostatic potential distribution with varying \( k_y \rho \) values.
Turbulence in finite beta LHD plasmas

Kinetic ballooning modes in a model configuration
Most unstable KBM has finite radial wavenumber

The most unstable KBM has finite radial wavenumber, which corresponds to finite $\theta_k$ in the ballooning representation.

$k_x=0$ mode is the most unstable KBM in CBC tokamak

A. Ishizawa, et.al., Nuclear Fusion, 053007 (2013)
The most unstable KBM has finite radial mode number (finite \( \theta_k \))

- The peak of the amplitude of electrostatic potential appears at the minimum of absolute value of magnetic drift.

\[ \phi \]
Nonlinear simulation results

• Zonal flow of KBM turbulence is much weaker than that of ITG turbulence.
• KBM turbulence is not efficient in the transport compared with ITG turbulence.

\[
\begin{align*}
\text{ITG} & \quad Q_i \approx 5n_0T_i\nu_{Ti}\rho_i^2 / L_n^2 \\
\text{KBM} & \quad Q_i \approx 3n_0T_i\nu_{Ti}\rho_i^2 / L_n^2
\end{align*}
\]

\[
\begin{align*}
\rho & \approx n_0 / T_i \\
\rho & \approx n_0 / \nu_{Ti}
\end{align*}
\]

\[
\begin{align*}
\Theta_{\text{ion}} & = 2.94 & \Theta_{\text{ele}} & = -1.04 & \Gamma_{\text{ion}} = \Gamma_{\text{ele}} & = 0.82 & Q_{\text{ion}} & = 4.98 & Q_{\text{ele}} & = 1.0 \\
\Theta_{\text{ion}} & = -0.01 & \Theta_{\text{ele}} & = 0.01 & \Gamma_{\text{ion}} = \Gamma_{\text{ele}} & = -0.02 & Q_{\text{ion}} & = -0.05 & Q_{\text{ele}} & = -0.03 \\
\Theta_{\text{ion}} & = 2.93 & \Theta_{\text{ele}} & = -1.03 & \Gamma_{\text{ion}} = \Gamma_{\text{ele}} & = 0.8 & Q_{\text{ion}} & = 4.93 & Q_{\text{ele}} & = 0.97
\end{align*}
\]

\[
\begin{align*}
\Theta_{\text{ion}} & = 1.94 & \Theta_{\text{ele}} & = -0.45 & \Gamma_{\text{ion}} = \Gamma_{\text{ele}} & = 0.37 & Q_{\text{ion}} & = 2.86 & Q_{\text{ele}} & = 0.47 \\
\Theta_{\text{ion}} & = -0.03 & \Theta_{\text{ele}} & = 0.22 & \Gamma_{\text{ion}} = \Gamma_{\text{ele}} & = 0.00 & Q_{\text{ion}} & = -0.02 & Q_{\text{ele}} & = 0.23 \\
\Theta_{\text{ion}} & = 1.91 & \Theta_{\text{ele}} & = -0.23 & \Gamma_{\text{ion}} = \Gamma_{\text{ele}} & = 0.37 & Q_{\text{ion}} & = 2.84 & Q_{\text{ele}} & = 0.70
\end{align*}
\]
New saturation mechanism of turbulence with weak zonal flow

- Most unstable KBM has an inclined mode structure
- Saturation of the KBM turbulence is caused by nonlinear interactions between inclined modes.
New saturation mechanism

- Saturation of the KBM turbulence is caused by mutual shearing between convection cells of inclined modes.
Entropy balance equation

\[
\frac{d}{dt}\left( \sum_s \delta S_s + \delta W_{es} + \delta W_{em} \right) = \sum_s \left( \frac{Q_s}{L_{Ts}} + \frac{T_s \Gamma_s}{L_{ps}} + D_s \right)
\]

\[
\delta S_s = \left\langle \sum_k \int \frac{T_s |\delta f_{sk}|^2}{2F_{Ms}} d^3v \right\rangle
\]

\[
\delta W_{em} = \left\langle \sum_s k_{\perp}^2 |\delta A_{\parallel k}|^2 \right\rangle
\]

\[
\Gamma_{es,s} = \text{Re} \left\langle \sum_k \delta n_s \frac{ik_y \delta \phi^*_k}{B} \right\rangle
\]

\[
\Gamma_{em,s} = \text{Re} \left\langle \sum_k \delta u_s \frac{ik_y \delta A^*_k}{B} \right\rangle
\]

\[
D_s = \nu_{ss} \left\langle \sum_k \int \left( \delta f_{sk} + \frac{q_s F_{sM}}{T_s} \phi_J J_{0s} \right)^* C(\delta f_{sk}) d^3v \right\rangle
\]

\[
\delta W_{es} = \left\langle \sum_k \left( \lambda^2_{Di} k_{\perp}^2 + \sum_s \frac{q_s^2}{T_s} (1 - \Gamma_{0s}) \right) |\delta \phi_k|^2 \right\rangle
\]

\[
Q_{es,s} = \text{Re} \left\langle \sum_k \left( \frac{\delta p_{\parallel s}}{2} + \frac{\delta p_{\perp s}}{2} - 5T_s \delta n_s \right) \frac{ik_y \delta \phi_k}{B} \right\rangle
\]

\[
Q_{em,s} = \text{Re} \left\langle \sum_k \left( \frac{\delta q_{\parallel s}}{2} + \delta q_{\perp s} \right) \frac{-ik_y \delta A^*_k}{B} \right\rangle
\]

Sugama Phys. Plasmas 2009
Saturation mechanism I

- Diagram of nonlinear entropy transfer in the Fourier space
- Saturation of the KBM turbulence is caused by nonlinear interactions between dominant unstable modes with finite radial wavenumbers

Entrophy transfer function

\[ T(k; k', k'') = \left\langle \int dv^3 \frac{T_s g_{sk}^*}{2F_{Ms}} \delta_{k, k'+k''} \mathbf{b} \cdot \mathbf{k}' \times \mathbf{k}'' \left( \chi_{sk} g_{sk} - g_{sk'} \chi_{sk'} \right) \right\rangle \]

\[ g_{sk} = f_{sk} + q_s \phi_k J_{0s} F_{Ms}, \quad \chi_{sk} = (\phi_k - v_{Ts} v_{//} A_{//}) J_{0s} \]
Saturation mechanism II

• The dominant KBM causes the transfer in the inclined direction and subsequently transforms the entropy from the other dominant KBM to higher Fourier modes, which are linearly stable.

• Hence, the growth of KBM is saturated by the nonlinear interactions of oppositely inclined convection cells through mutual shearing.
Summary

• GK analysis of finite-beta LHD plasmas

• Kinetic ballooning turbulence
  – Inclined mode (finite theta_k) is most unstable.
  – Weak zonal flow
  – Small efficiency in transport

• A new saturation mechanism, which is the shearing between oppositely inclined modes, is relevant in the saturation of high-beta turbulence in the presence of three-dimensionality.